14.74 Problem Set #5
Due Wednesday, 4/20, in class by 1:15pm. Problem sets will receive one grade late penalty between 1:15 and 2:30pm. No problem sets accepted after 2:30pm.

1. **Sharecropping** (When asked to solve, please write the steps and the equations clearly and explain what they mean – it is not enough to copy the final answers from the lecture notes). Consider the following setting
   - Tenant farms the land and applies effort, \( e \)
   - Tenant’s outside option is 0, and tenant has wealth \( w \)
   - The landlord cannot observe \( e \)
   - Both the tenant and the landlord are risk-neutral
   - Effort is costly, the cost of effort is \( .5ce^2 \)
   - Two things can happen:
     - With probability \( e \): Output is \( H \)
     - With probability \( 1-e \): Output is 0 (zero)
   - The tenant and the landlord write a contract which specifies a payment from the landlord to the tenant
     - Payment of \( h \) if output is \( H \)
     - Payment of \( l \) if output is 0
   - Tenant and landlord maximize their expected utility, which is equal to expected income for the landlord, and expected income minus cost of effort for the tenant.

1) What do \( l \) and \( h \) look like in a sharecropping contract? In a rental contract? (Give the general form of \( l \) and \( h \)).

2) **Optimal effort** (first best):
   a) What problem do you need to solve to determine the optimal level of effort (the one that would be chosen if there were no contractual difficulties)?
   b) Solve the problem: what is \( e^{FB} \), the first best level of effort?

3) **Tenant’s choice of effort**
   a) Given a contract that specifies \( h \) and \( l \), what is the tenant’s effort? (Set up the tenant’s utility maximization problem and solve the first order conditions).
   b) What class of contract will lead the tenant to choose the FB level of effort, \( e^{FB} \)?

4) **Optimal Contract without limited liability**
   a) Assume there is no limited liability. The landlord maximizes his income. What contract will the landlord propose to the tenant? Calculate the rent that the landlord charges
   b) What is the tenant’s utility under this contract?

5) **Optimal contract with limited liability** – the landlord cannot take more than the tenant’s wealth \( (l \geq -w) \)
   a) Is the tenant’s problem and effort choice given \( h \) and \( l \) changed?
   b) What is \( l \) chosen by the landlord?
   c) How does the landlord choose \( h \)? What condition on \( H, C, w \) is needed such that the tenant is willing to participate in this contract (remember that we assume in (b) the limited liability constraint binds, solve and plug the solution into the other constraint to find the condition for this constraint to be satisfied).
   d) What is the tenant’s effort under this contract? \( e^{LL} \): effort under limited liability)? How does it compare to \( e^{FB} \) and why?
e) What is the tenant’s utility under this contract? How does it compare to the utility under a rental contract?

f) Now think of a market with many tenants and landlords. Suppose different plots of land have different fertility levels and therefore H varies (and has a distribution G(H)). Each landlord has 1 plot of land and all plots of land are the same size. Let there be a large number of potential tenants whose w varies (distribution F(w)) and there be more potential tenants than there are plots of land, so that some tenants will earn their outside option of 0. The allocation of tenants to landlords is generated by a competitive market where landlords compete for the tenants they like the best. What would the matching look like in equilibrium, i.e. landlords with low (high) H are matched with which kind of tenants? No need for closed form solutions here, but justify your arguments.